

# INTEGRABLE SYSTEMS FROM MEMBRANES ON $AdS_4 \times S^7$

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We describe how Neumann and Neumann-Rosochatius type integrable systems, as well as the continuous limit of the  $SU(2)$  integrable spin chain, can be obtained from membranes on  $AdS_4 \times S^7$  background, in the framework of AdS/CFT correspondence.

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## 1 Introduction

The 2-branes (membranes) and 5-branes are the fundamental dynamical objects in the eleven dimensional  $M$ -theory, which is the strong coupling limit of the five superstring theories in ten dimensions, and whose low energy field theory limit is the eleven dimensional supergravity.

It is known that large class of classical string solutions in the type IIB  $AdS_5 \times S^5$  background is related to the Neumann and Neumann-Rosochatius integrable systems, including recently discovered spiky strings and giant magnons [1]. It is also interesting if these integrable systems can be associated with some membrane configurations in  $M$ -theory. We explain here how this can be achieved by considering membrane embedding in  $AdS_4 \times S^7$  solution of  $M$ -theory, with the desired properties.

On the other hand, we will show the existence of membrane configurations in  $AdS_4 \times S^7$ , which correspond to the continuous limit of the  $SU(2)$  integrable spin chain, arising in  $\mathcal{N} = 4$  SYM in four dimensions, dual to strings in  $AdS_5 \times S^5$  [2].

## 2 Membranes on $AdS_4 \times S^7$

We start with the following membrane action

$$S = \int d^3\xi \mathcal{L} \tag{2.1}$$

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$$= \int d^3\xi \left\{ \frac{1}{4\lambda^0} [G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} - (2\lambda^0 T_2)^2 \det G_{ij}] + T_2 C_{012} \right\},$$

where

$$\begin{aligned} G_{mn} &= g_{MN}(X) \partial_m X^M \partial_n X^N, \quad C_{012} = c_{MNP}(X) \partial_0 X^M \partial_1 X^N \partial_2 X^P, \\ \partial_m &= \partial / \partial \xi^m, \quad m = (0, i) = (0, 1, 2), \\ (\xi^0, \xi^1, \xi^2) &= (\tau, \sigma_1, \sigma_2), \quad M = (0, 1, \dots, 10), \end{aligned}$$

are the fields induced on the membrane worldvolume from the background metric  $g_{MN}$  and the background 3-form gauge field  $c_{MNP}$ ,  $\lambda^m$  are Lagrange multipliers,  $x^M = X^M(\xi)$  are the membrane embedding coordinates, and  $T_2$  is its tension. As shown in [3], the above action is classically equivalent to the Nambu-Goto type action

$$S^{NG} = -T_2 \int d^3\xi \left( \sqrt{-\det G_{mn}} - \frac{1}{6} \varepsilon^{mnp} \partial_m X^M \partial_n X^N \partial_p X^P c_{MNP} \right)$$

and to the Polyakov type action

$$S^P = -\frac{T_2}{2} \int d^3\xi \left[ \sqrt{-\gamma} (\gamma^{mn} G_{mn} - 1) - \frac{1}{3} \varepsilon^{mnp} \partial_m X^M \partial_n X^N \partial_p X^P c_{MNP} \right],$$

where  $\gamma^{mn}$  is the auxiliary worldvolume metric and  $\gamma = \det \gamma_{mn}$ . In addition, the action (2.1) gives a unified description for the tensile and tensionless membranes.

The equations of motion for the Lagrange multipliers  $\lambda^m$  generate the constraints

$$G_{00} - 2\lambda^j G_{0j} + \lambda^i \lambda^j G_{ij} + (2\lambda^0 T_2)^2 \det G_{ij} = 0, \quad (2.2)$$

$$G_{0j} - \lambda^i G_{ij} = 0. \quad (2.3)$$

Further on, we will work in diagonal worldvolume gauge  $\lambda^i = 0$ , in which the action (2.1) and the constraints (2.2), (2.3) simplify to

$$S_M = \int d^3\xi \mathcal{L}_M = \int d^3\xi \left\{ \frac{1}{4\lambda^0} [G_{00} - (2\lambda^0 T_2)^2 \det G_{ij}] + T_2 C_{012} \right\}, \quad (2.4)$$

$$G_{00} + (2\lambda^0 T_2)^2 \det G_{ij} = 0, \quad (2.5)$$

$$G_{0i} = 0. \quad (2.6)$$

Let us note that the action (2.4) and the constraints (2.5), (2.6) *coincide* with the usually used gauge fixed Polyakov type action and constraints after the following identification of the parameters  $2\lambda^0 T_2 = L = \text{const}$  (see for instance [4]).

Searching for membrane configurations in  $AdS_4 \times S^7$ , which correspond to the Neumann or Neumann-Rosochatius integrable systems, we should first eliminate the membrane interaction with the background 3-form field on  $AdS_4$ , to ensure more close analogy with the strings on  $AdS_5 \times S^5$ . To make our choice, let us write down the background. It can be parameterized as follows

$$\begin{aligned} ds^2 &= (2l_p \mathcal{R})^2 \left[ -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho (d\alpha^2 + \sin^2 \alpha d\beta^2) + 4d\Omega_7^2 \right], \\ d\Omega_7^2 &= d\psi_1^2 + \cos^2 \psi_1 d\varphi_1^2 \\ &+ \sin^2 \psi_1 \left[ d\psi_2^2 + \cos^2 \psi_2 d\varphi_2^2 + \sin^2 \psi_2 (d\psi_3^2 + \cos^2 \psi_3 d\varphi_3^2 + \sin^2 \psi_3 d\varphi_4^2) \right], \\ c_{(3)} &= (2l_p \mathcal{R})^3 \sinh^3 \rho \sin \alpha dt \wedge d\alpha \wedge d\beta. \end{aligned}$$

Since we want the membrane to have nonzero conserved energy and spin on  $AdS$ , one possible choice, for which the interaction with the  $c_{(3)}$  field disappears, is to fix the angle  $\alpha$ :  $\alpha = \alpha_0 = \text{const}$ . The metric of the corresponding subspace of  $AdS_4$  is

$$ds_{sub}^2 = (2l_p \mathcal{R})^2 \left[ -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d(\beta \sin \alpha_0)^2 \right].$$

The appropriate membrane embedding into  $ds_{sub}^2$  and  $S^7$  is

$$\begin{aligned} Z_\mu &= 2l_p \mathcal{R} r_\mu(\xi^m) e^{i\phi_\mu(\xi^m)}, & \mu &= (0, 1), & \phi_\mu &= (\phi_0, \phi_1) = (t, \beta \sin \alpha_0), \\ W_a &= 4l_p \mathcal{R} r_a(\xi^m) e^{i\varphi_a(\xi^m)}, & a &= (1, 2, 3, 4), \end{aligned}$$

where  $r_\mu$  and  $r_a$  are real functions of  $\xi^m$ , while  $\phi_\mu$  and  $\varphi_a$  are the isometric coordinates on which the background metric does not depend. The six complex coordinates  $Z_\mu$ ,  $W_a$  are restricted by the two real embedding constraints

$$\eta^{\mu\nu} Z_\mu \bar{Z}_\nu + (2l_p \mathcal{R})^2 = 0, \quad \eta^{\mu\nu} = (-1, 1), \quad \delta_{ab} W_a \bar{W}_b - (4l_p \mathcal{R})^2 = 0,$$

or equivalently

$$\eta^{\mu\nu} r_\mu r_\nu + 1 = 0, \quad \delta_{ab} r_a r_b - 1 = 0.$$

The coordinates  $r_\mu$ ,  $r_a$  are connected to the initial coordinates, on which the background depends, through the equalities

$$\begin{aligned} r_0 &= \cosh \rho, & r_1 &= \sinh \rho, \\ r_1 &= \cos \psi_1, & r_2 &= \sin \psi_1 \cos \psi_2, \\ r_3 &= \sin \psi_1 \sin \psi_2 \cos \psi_3, & r_4 &= \sin \psi_1 \sin \psi_2 \sin \psi_3. \end{aligned}$$

For the embedding described above, the induced metric is given by

$$\begin{aligned} G_{mn} &= \eta^{\mu\nu} \partial_{(m} Z_\mu \partial_{n)} \bar{Z}_\nu + \delta_{ab} \partial_{(m} W_a \partial_{n)} \bar{W}_b = \\ &= (2l_p \mathcal{R})^2 \left[ \sum_{\mu, \nu=0}^1 \eta^{\mu\nu} \left( \partial_m r_\mu \partial_n r_\nu + r_\mu^2 \partial_m \phi_\mu \partial_n \phi_\nu \right) + 4 \sum_{a=1}^4 \left( \partial_m r_a \partial_n r_a + r_a^2 \partial_m \varphi_a \partial_n \varphi_a \right) \right]. \end{aligned} \tag{2.7}$$

Correspondingly, the membrane Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_M + \Lambda_A (\eta^{\mu\nu} r_\mu r_\nu + 1) + \Lambda_S (\delta_{ab} r_a r_b - 1),$$

where  $\Lambda_A$  and  $\Lambda_S$  are Lagrange multipliers.

## 2.1 Neumann and Neumann-Rosochatius integrable systems from membranes

Let us consider the following particular case of the above membrane embedding [5]

$$\begin{aligned} Z_0 &= 2l_p \mathcal{R} e^{i\kappa\tau}, \quad Z_1 = 0, & W_a &= 4l_p \mathcal{R} r_a(\xi, \eta) e^{i[\omega_a \tau + \mu_a(\xi, \eta)]}, \\ \xi &= \alpha \sigma_1 + \beta \tau, \quad \eta = \gamma \sigma_2 + \delta \tau, \end{aligned} \tag{2.8}$$

which implies

$$r_0 = 1, \quad r_1 = 0, \quad \phi_0 = t = \kappa\tau, \quad \varphi_a(\xi^m) = \varphi_a(\tau, \sigma_1, \sigma_2) = \omega_a\tau + \mu_a(\xi, \eta). \quad (2.9)$$

Here  $\kappa, \omega_a, \alpha, \beta, \gamma, \delta$  are parameters, whereas  $r_a(\xi, \eta), \mu_a(\xi, \eta)$  are arbitrary functions. As a consequence, the embedding constraint  $\eta^{\mu\nu}r_\mu r_\nu + 1 = 0$  is satisfied identically. For this ansatz, the membrane Lagrangian takes the form ( $\partial_\xi = \partial/\partial\xi, \partial_\eta = \partial/\partial\eta$ )

$$\begin{aligned} \mathcal{L} = & -\frac{(4l_p\mathcal{R})^2}{4\lambda^0} \left\{ \left(8\lambda^0 T_2 l_p \mathcal{R} \alpha \gamma\right)^2 \sum_{a < b=1}^4 \left[ (\partial_\xi r_a \partial_\eta r_b - \partial_\eta r_a \partial_\xi r_b)^2 \right. \right. \\ & + (\partial_\xi r_a \partial_\eta \mu_b - \partial_\eta r_a \partial_\xi \mu_b)^2 r_b^2 + (\partial_\xi \mu_a \partial_\eta r_b - \partial_\eta \mu_a \partial_\xi r_b)^2 r_a^2 \\ & \left. \left. + (\partial_\xi \mu_a \partial_\eta \mu_b - \partial_\eta \mu_a \partial_\xi \mu_b)^2 r_a^2 r_b^2 \right] \right. \\ & + \sum_{a=1}^4 \left[ \left(8\lambda^0 T_2 l_p \mathcal{R} \alpha \gamma\right)^2 (\partial_\xi r_a \partial_\eta \mu_a - \partial_\eta r_a \partial_\xi \mu_a)^2 - (\beta \partial_\xi \mu_a + \delta \partial_\eta \mu_a + \omega_a)^2 \right] r_a^2 \\ & \left. - \sum_{a=1}^4 (\beta \partial_\xi r_a + \delta \partial_\eta r_a)^2 + (\kappa/2)^2 \right\} + \Lambda_S \left( \sum_{a=1}^4 r_a^2 - 1 \right). \end{aligned}$$

Now, we make the choice

$$\begin{aligned} r_1 &= r_1(\xi), \quad r_2 = r_2(\xi), \quad \omega_3 = \pm\omega_4 = \omega, \\ r_3 &= r_3(\eta) = \epsilon \sin(b\eta + c), \quad r_4 = r_4(\eta) = \epsilon \cos(b\eta + c), \\ \mu_1 &= \mu_1(\xi), \quad \mu_2 = \mu_2(\xi), \quad \mu_3, \mu_4 = \text{constants}, \end{aligned}$$

and receive (prime is used for  $d/d\xi$ )

$$\begin{aligned} \mathcal{L} = & -\frac{(4l_p\mathcal{R})^2}{4\lambda^0} \left\{ \sum_{a=1}^2 \left[ (A^2 - \beta^2) r_a'^2 + (A^2 - \beta^2) r_a^2 \left( \mu_a' - \frac{\beta\omega_a}{A^2 - \beta^2} \right)^2 - \frac{A^2}{A^2 - \beta^2} \omega_a^2 r_a^2 \right] \right. \\ & \left. + (\kappa/2)^2 - \epsilon^2(\omega^2 + b^2\delta^2) \right\} + \Lambda_S \left[ \sum_{a=1}^2 r_a^2 - (1 - \epsilon^2) \right], \end{aligned}$$

where  $A^2 \equiv (8\lambda^0 T_2 l_p \mathcal{R} \epsilon b \alpha \gamma)^2$ . A single time integration of the equations of motion for  $\mu_a$  following from the above Lagrangian gives

$$\mu_a' = \frac{1}{A^2 - \beta^2} \left( \frac{C_a}{r_a^2} + \beta\omega_a \right),$$

where  $C_a$  are arbitrary constants. Taking this into account, one obtains the following effective Lagrangian for the coordinates  $r_a(\xi)$

$$\begin{aligned} L = & \frac{(4l_p\mathcal{R})^2}{4\lambda^0} \sum_{a=1}^2 \left[ (A^2 - \beta^2) r_a'^2 - \frac{1}{A^2 - \beta^2} \frac{C_a^2}{r_a^2} - \frac{A^2}{A^2 - \beta^2} \omega_a^2 r_a^2 \right] \\ & + \Lambda_S \left[ \sum_{a=1}^2 r_a^2 - (1 - \epsilon^2) \right]. \end{aligned}$$

This Lagrangian in full analogy with the string considerations corresponds to particular case of the  $n$ -dimensional *Neumann-Rosochatius integrable system*. For  $C_a = 0$  one obtains *Neumann integrable system*, which describes two-dimensional harmonic oscillator, constrained to remain on a circle of radius  $\sqrt{1 - \epsilon^2}$ .

Let us write down the three constraints (2.5), (2.6) for the present case. To achieve more close correspondence with the string on  $AdS_5 \times S^5$ , we want the third one,  $G_{02} = 0$ , to be satisfied identically. To this end, since  $G_{02} \sim (ab)^2 \gamma \delta$ , we set  $\delta = 0$ , i.e.  $\eta = \gamma \sigma_2$ . Then, the first two constraints give

$$\begin{aligned} \sum_{a=1}^2 \left[ (A^2 - \beta^2) r_a'^2 + \frac{1}{A^2 - \beta^2} \frac{C_a^2}{r_a^2} + \frac{A^2}{A^2 - \beta^2} \omega_a^2 r_a^2 \right] &= \frac{A^2 + \beta^2}{A^2 - \beta^2} \left[ (\kappa/2)^2 - (\epsilon\omega)^2 \right], \\ \sum_{a=1}^2 \omega_a C_a + \beta \left[ (\kappa/2)^2 - (\epsilon\omega)^2 \right] &= 0. \end{aligned} \quad (2.10)$$

## 2.2 Energy and angular momenta

Due to the background isometries, there exist global conserved charges. In our case, the background does not depend on  $\phi_0 = t$  and  $\varphi_a$ . Therefore, the corresponding conserved quantities are the membrane energy  $E$  and four angular momenta  $J_a$ , given as spatial integrals of the conjugated to these coordinates momentum densities

$$E = - \int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 t)}, \quad J_a = \int d^2\sigma \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_a)}, \quad a = 1, 2, 3, 4.$$

$E$  and  $J_a$  can be computed by using the expression (2.7) for the induced metric and the ansatz (2.8), (2.9).

In order to reproduce the string case, we can set  $\omega = 0$ , and thus  $J_3 = J_4 = 0$ . The energy and the other two angular momenta are given by

$$E = \frac{4\pi(l_p \mathcal{R})^2 \kappa}{\lambda^0 \alpha} \int d\xi, \quad J_a = \frac{\pi(4l_p \mathcal{R})^2}{\lambda^0 \alpha (A^2 - \beta^2)} \int d\xi \left( \beta C_a + A^2 \omega_a r_a^2 \right), \quad a = 1, 2.$$

From here, by using the constraints (2.10), one obtains the energy-charge relation

$$\frac{4}{A^2 - \beta^2} \left[ A^2(1 - \epsilon^2) + \beta \sum_{a=1}^2 \frac{C_a}{\omega_a} \right] \frac{E}{\kappa} = \sum_{a=1}^2 \frac{J_a}{\omega_a},$$

in full analogy with the string case. Namely, for strings on  $AdS_5 \times S^5$ , the result in conformal gauge is [1]

$$\frac{1}{\alpha^2 - \beta^2} \left( \alpha^2 + \beta \sum_a \frac{C_a}{\omega_a} \right) \frac{E}{\kappa} = \sum_a \frac{J_a}{\omega_a}.$$

Let us point out that the membrane configuration considered here corresponds exactly to the string embedding in the  $R \times S^5$  subspace of  $AdS_5 \times S^5$  solution of type IIB string theory, which is known to lead the Neumann and Neumann-Rosochatius dynamical systems [1], including recently discovered giant magnon and spiky string configurations.

### 3 $SU(2)$ spin chain from membrane

One of the predictions of AdS/CFT duality is that the string theory on  $AdS_5 \times S^5$  should be dual to  $\mathcal{N} = 4$  SYM theory in four dimensions. The spectrum of the string states and of the operators in SYM should be the same. The first checks of this conjecture *beyond* the supergravity approximation revealed that there exist string configurations, whose energies in the semiclassical limit are related to the anomalous dimensions of certain gauge invariant operators in the planar SYM. On the field theory side, it was found that the corresponding dilatation operator is connected to the Hamiltonian of integrable Heisenberg spin chain. On the other hand, it was established that there is agreement at the level of actions between the continuous limit of the  $SU(2)$  spin chain arising in  $\mathcal{N} = 4$  SYM theory and a certain limit of the string action in  $AdS_5 \times S^5$  background. Shortly after, it was shown that such equivalence also holds for the  $SU(3)$  and  $SL(2)$  cases.

Here, we are interested in answering the question: is it possible to reproduce this type of string/spin chain correspondence from membranes on eleven dimensional curved backgrounds? It turns out that the answer is positive at least for the case of M2-branes on  $AdS_4 \times S^7$ , as we will show below [6].

We will use our initial membrane embedding and fix

$$Z_0 = 2l_p \mathcal{R} e^{i\kappa\tau}, \quad Z_1 = 0,$$

which implies  $r_0 = 1$ ,  $r_1 = 0$ ,  $\phi_0 = t = \kappa\tau$ . Let us now introduce new coordinates by setting

$$(\varphi_1, \varphi_2, \varphi_3, \varphi_4) = \left( \frac{\kappa}{2}\tau + \alpha + \varphi, \frac{\kappa}{2}\tau + \alpha - \varphi, \frac{\kappa}{2}\tau + \alpha + \phi, \frac{\kappa}{2}\tau + \alpha - \tilde{\phi} \right)$$

and take the limit  $\kappa \rightarrow \infty$ ,  $\partial_0 \rightarrow 0$ ,  $\kappa\partial_0$  - finite. In this limit, we obtain the following expression for the membrane Lagrangian

$$\begin{aligned} \mathcal{L} = & \frac{(2l_p \mathcal{R})^2}{\lambda^0} \kappa \left( \partial_0 \alpha + \sum_{k=1}^3 \nu_k \partial_0 \rho_k \right) - \lambda^0 T_2^2 (4l_p \mathcal{R})^4 \left\{ \sum_{a < b=1}^4 (\partial_1 r_a \partial_2 r_b - \partial_2 r_a \partial_1 r_b)^2 \right. \\ & + \sum_{a=1}^4 \sum_{k=1}^3 \mu_k (\partial_1 r_a \partial_2 \rho_k - \partial_2 r_a \partial_1 \rho_k)^2 - \sum_{a=1}^4 \left( \partial_1 r_a \sum_{k=1}^3 \nu_k \partial_2 \rho_k - \partial_2 r_a \sum_{k=1}^3 \nu_k \partial_1 \rho_k \right)^2 \\ & + \sum_{k < n=1}^3 \mu_k \mu_n (\partial_1 \rho_k \partial_2 \rho_n - \partial_2 \rho_k \partial_1 \rho_n)^2 \\ & \left. - \sum_{k=1}^3 \mu_k \left( \partial_1 \rho_k \sum_{n=1}^3 \nu_n \partial_2 \rho_n - \partial_2 \rho_k \sum_{n=1}^3 \nu_n \partial_1 \rho_n \right)^2 \right\} + \Lambda_S \left( \sum_{a=1}^4 r_a^2 - 1 \right), \end{aligned}$$

where

$$(\mu_1, \mu_2, \mu_3) = (r_1^2 + r_2^2, r_3^2, r_4^2), (\nu_1, \nu_2, \nu_3) = (r_1^2 - r_2^2, r_3^2, -r_4^2), (\rho_1, \rho_2, \rho_3) = (\varphi, \phi, \tilde{\phi}).$$

Now, we are ready to face our main problem: how to reduce this Lagrangian to the one corresponding to the thermodynamic limit of spin chain, *without shrinking the membrane*

to string? We propose the following solution of this task:

$$\begin{aligned}\alpha &= \alpha(\tau, \sigma_1), & r_1 &= r_1(\tau, \sigma_1), & r_2 &= r_2(\tau, \sigma_1), \\ r_3 &= r_3(\tau, \sigma_2) = \epsilon \sin[b\sigma_2 + c(\tau)], & r_4 &= r_4(\tau, \sigma_2) = \epsilon \cos[b\sigma_2 + c(\tau)], \\ \varphi &= \varphi(\tau, \sigma_1), & \epsilon, b, \phi, \tilde{\phi} &= \text{constants}, & \epsilon^2 &< 1.\end{aligned}$$

These restrictions lead to

$$\begin{aligned}\mathcal{L} &= \frac{(2l_p \mathcal{R})^2}{\lambda^0} \kappa \left[ \partial_0 \alpha + (r_1^2 - r_2^2) \partial_0 \varphi \right] - \lambda^0 (\epsilon b T_2)^2 (4l_p \mathcal{R})^4 \left\{ \sum_{a=1}^2 (\partial_1 r_a)^2 \right. \\ &\quad \left. + \left[ (r_1^2 + r_2^2) - (r_1^2 - r_2^2)^2 \right] (\partial_1 \varphi)^2 \right\} + \Lambda_S \left[ \sum_{a=1}^2 r_a^2 - (1 - \epsilon^2) \right].\end{aligned}$$

If we introduce the parametrization

$$r_1 = (1 - \epsilon^2)^{1/2} \cos \psi, \quad r_2 = (1 - \epsilon^2)^{1/2} \sin \psi,$$

the new variable  $\tilde{\alpha} = \alpha / (1 - \epsilon^2)$ , and take the limit  $\epsilon^2 \rightarrow 0$  neglecting the terms of order higher than  $\epsilon^2$ , we will receive

$$\frac{\mathcal{L}}{1 - \epsilon^2} = \frac{(2l_p \mathcal{R})^2}{\lambda^0} \kappa \left[ \partial_0 \tilde{\alpha} + \cos(2\psi) \partial_0 \varphi \right] - \lambda^0 (\epsilon b T_2)^2 (4l_p \mathcal{R})^4 \left[ (\partial_1 \psi)^2 + \sin^2(2\psi) (\partial_1 \varphi)^2 \right].$$

As for the membrane action corresponding to the above Lagrangian, it can be represented in the form

$$S_M = \frac{\mathcal{J}}{2\pi} \int dt d\sigma \left[ \partial_t \tilde{\alpha} + \cos(2\psi) \partial_t \varphi \right] - \frac{\tilde{\lambda}}{4\pi \mathcal{J}} \int dt d\sigma \left[ (\partial_\sigma \psi)^2 + \sin^2(2\psi) (\partial_\sigma \varphi)^2 \right],$$

where  $\mathcal{J}$  is the angular momentum conjugated to  $\tilde{\alpha}$ ,  $t = \kappa \tau$  and

$$\tilde{\lambda} = 2^{15} (\pi^2 \epsilon b T_2)^2 (l_p \mathcal{R})^6.$$

This action corresponds to the thermodynamic limit of *SU(2) integrable spin chain* [2].

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